A NOTE ON VERTICAL MOTIONS IN THE REGION OF THE ANTARCTIC CIRCUMPOLAR CURRENT

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1. INTRODUCTION

Many discussions on the nature of the Antarctic Polar Front (Antarctic Convergence) with Dr. Wexler, in connection with his work (Wexler [5]), stimulated the author's interest in these problems. Generally, it is agreed that relatively strong vertical motions exist in the region of the Antarctic Circumpolar Current. These may be induced by the wind distribution which, however, would directly influence only a relatively shallow layer. Inasmuch as the polar front can be delineated also at great depth (Deacon [1]), and in view of its relatively fixed position (Deacon [2]), it is felt that the primary reason or at least one of the more important causes for its existence must be sought in the dynamics of the Antarctic Circumpolar Current. Several studies in this direction have been made recently (Wyrtki [6]; Ostapoff [4]).

This note tries to show what kind of cross circulation can be expected for certain specified velocity distributions of the Antarctic Circumpolar Current under conditions of predominance of lateral momentum exchange. Although the wind stress does not appear explicitly in this model, its effect has been tacitly assumed by the requirement that the zonal velocity profile be maintained unchanged, while momentum is continually diffused out from the jet.

2. WORKING MODELS

The following assumptions (Ostapoff [4]) are introduced into the hydrodynamic equations: stationary conditions, no non-linear interaction, eddy momentum exchange with constant coefficients, and no pressure gradient downstream of the basic current. In addition it is assumed here that the Coriolis parameter is constant. With these simplifications the equations become

$$-f\rho v = A_v u_{zz} + A_h u_{yy} = F(y, z) \qquad (1)$$

$$[\rho v]_y + [\rho w]_z = 0 \qquad (2)$$

where u, v, w denote the velocity components in the x, y, z directions, respectively; ρ denotes the density of water, f the Coriolis parameter, and A_v, A_h the vertical and horizontal exchange coefficients for momentum, respectively. The subscripts denote partial differentiation with respect to the indicated variable.

Differentiating (1) with respect to y (f=const.) and utilizing (2) yields

$$[\rho w]_z = f^{-1} F_y \qquad (3)$$

Integration of (3) with $\rho w=0$ at $z=0$ gives an equation for the vertical motion

$$\rho w|_{z=h} = f^{-1} \int_0^h F_y dz \qquad (4)$$

Equation (2) permits the introduction of a stream function

$$\rho v = \psi_z$$

$$\rho w = -\psi_y \qquad (5)$$

Thus, equations (5), (4), and (1) combined lead to

$$\psi(y, z) = -f^{-1} \int F dz - f^{-1} \int \int F_y dz dy$$

or, provided the order of integration in the double integral can be reversed,

$$\psi(y, z) = -2f^{-1} \int F dz \qquad (6)$$

Finally, the u-distribution remains to be specified.

Case 1.—The simplest assumption about the zonal horizontal velocity component is that it has a jet-like distribution in the horizontal with a single maximum and a cosine dependency on depth. Such a distribution can be written as

$$u(y, z) = u_0 \cos\left(\frac{\pi z}{2d}\right) [a + by^2]^{-1} \qquad (7)$$

and equation (6) becomes

$$\psi = -\frac{4d}{\pi f} u_0 \sin\left(\frac{\pi z}{2d}\right) \{A_h 8b^2y^2[a+by^2]^{-3} - A_h 2b[a+by^2]^{-2} - \pi^2 A_v[4d^2(a+by^2)]^{-1}\} \qquad (8)$$

The following numerical values have been used to evaluate equation (8):

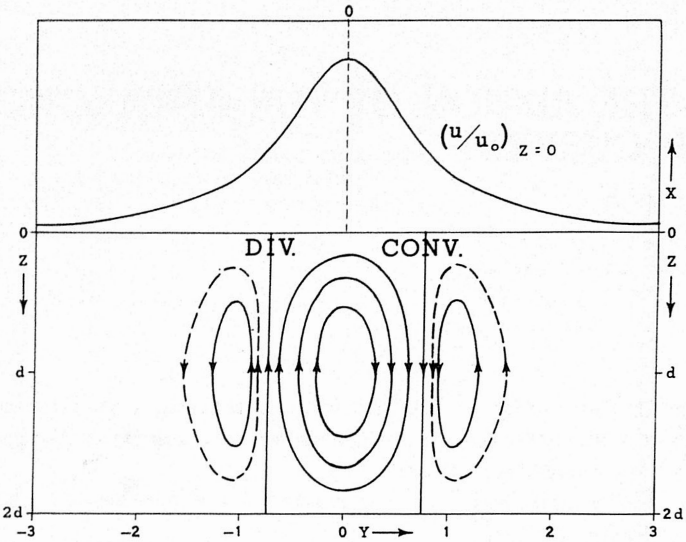

FIGURE 1.—Schematic presentation of streamlines for case 1.

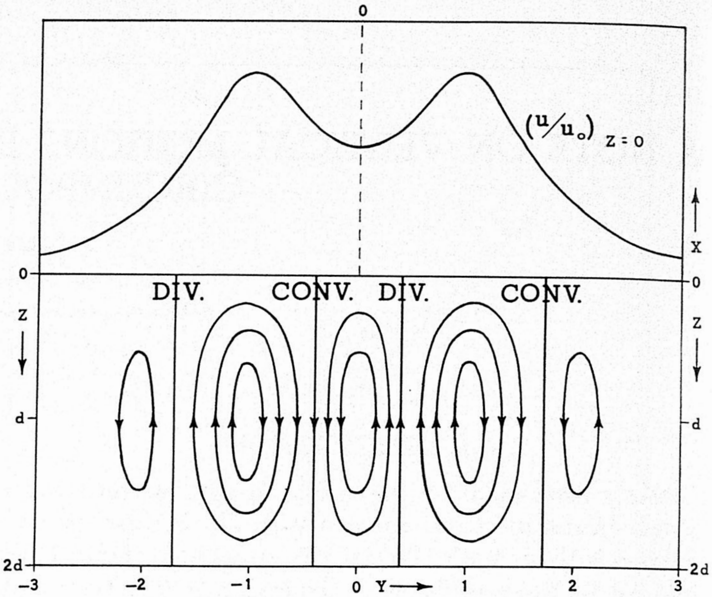

FIGURE 2.—Schematic presentation of streamlines for case 2.

$$a=1,\ b=2,\ d=2\times10^{-2},\ A_h/A_v=10^6.$$

In figure 1 the results are plotted for the quantity $\pi f(4du_0)^{-1}\psi$ in arbitrary units. The top portion of figure 1 shows the assumed u-component at the surface ($z=0$). The lower portion shows the corresponding cross circulation pattern which would result if the terms considered were the only acting forces. A three-celled cross circulation is apparent with the strongest downward motion in the "southern" portion of the u-profile and the strongest upward motion in the "northern" portion. This general pattern is substantially similar to the one presented earlier (Ostapoff [4]) which was calculated numerically on the basis of a geostrophic velocity distribution.

It should be noted that the circulation (as shown in fig. 1) in the upper layer ($0\leq z\leq d$) is identical to Wyrtki's case of "weak west winds" (Wyrtki [7]), if the u-maximum coincides with the inflection point in the topography of the sea surface.

The cross circulation pattern as presented in figure 1 (and for that matter also in fig. 2) depends on the ratio of the momentum exchange coefficients. In our example we have chosen perhaps a rather high horizontal exchange coefficient (for $A_v=10$, $A_h=10^7$). Reducing A_h by one order of magnitude does not change substantially the pattern presented in figure 1. However, for a ratio $A_h/A_v=10^4$ (i.e. $A_v=10$, $A_h=10^5$) there is a tendency toward a one-celled cross circulation centered around the u-maximum and possessing the same sense of rotation as the center cell in figure 1. Thus, the upward motion south of the convergence (fig. 1) becomes apparent only for relatively high lateral momentum diffusion.

Case 2.—The second case investigated may be of interest because the basic current may exhibit a double maximum or streakiness with several maxima. Also, it was suggested that the wind distribution may have a double maximum (Ivanov [3]), from which oceanic frontal zones have been derived. This case may be described by

$$u(y,z)=u_0\cos\left(\frac{\pi z}{2d}\right)\{[a+b(y-c)^2]^{-1}+[a+b(y+c)^2]^{-1}\} \tag{9}$$

and equation (8) becomes

$$\begin{aligned}\psi=-\frac{4d}{\pi f}u_0\sin\left(\frac{\pi z}{2d}\right)\{&A_h8b^2[y-c]^2[a+b(y-c)^2]^{-3}\\&+A_h8b^2[y+c]^2[a+b(y+c)^2]^{-3}-A_h2b[a+b(y-c)^2]^{-2}\\&-A_h2b[a+b(y+c)^2]^{-2}-\pi^2A_v(4d^2)^{-1}[a+b(y-c)^2]^{-1}\\&-\pi^2A_v(4d^2)^{-1}[a+b(y+c)^2]^{-1}\}\end{aligned} \tag{10}$$

The quantity $\pi f(4du_o)^{-1}\psi$ is plotted on a relative scale in figure 2 with $c=1$ and the other constants as in Case 1.

As evident from figure 2, the simple convergence-divergence scheme is replaced by a convergence-divergence-convergence-divergence pattern somewhat similar to the one proposed by Ivanov [3] on the basis of wind distribution. Thus, it could be expected that any current with multiple current axes will have a tendency to show as many convergence-divergences as current maxima. This result was anticipated by Wexler [5] on the basis of atmospheric streakiness and multiple jet-stream structures.

He concluded that, "The cause for these highly organized concentrations of momentum is not known but by analogy one would expect it to be present in a well-marked oceanic "jet-stream" in the Antarctic Circumpolar Current (ACC). Since the examples of streakiness cited above all have a system of horizontal divergences and convergences arranged along the axis of the current, it would appear that this should also be characteristic of the ACC."

3. CONCLUSION

Using a very simple model it has been shown schematically how regions of convergence and divergence may develop in a current with different horizontal profiles. Upon a current with a single velocity maximum a torque is superimposed in such a way that in the Southern Hemisphere downward motion results in the right-hand portion of the stream and upward motion in the left-hand portion (and vice versa in the Northern Hemisphere). Variation of the Coriolis parameter makes the poleward portions more narrow, thus intensifying there the vertical flow.

A current system with several velocity maxima appears to have a tendency to form a system of convergence-divergences leading to a characteristic streakiness.

ACKNOWLEDGMENT

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